

RESEARCH ARTICLE

cocor: A Comprehensive Solution for the Statistical Comparison of Correlations

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Abstract

A valid comparison of the magnitude of two correlations requires researchers to directly contrast the correlations using an appropriate statistical test. In many popular statistics packages, however, tests for the significance of the difference between correlations are missing. To close this gap, we introduce `cocor`, a free software package for the R programming language. The `cocor` package covers a broad range of tests including the comparisons of independent and dependent correlations with either overlapping or nonoverlapping variables. The package also includes an implementation of Zou's confidence interval for all of these comparisons. The platform independent `cocor` package enhances the R statistical computing environment and is available for scripting. Two different graphical user interfaces—a plugin for Rkward and a web interface—make `cocor` a convenient and user-friendly tool.



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Data Availability Statement: The cocor R package can be downloaded from <http://cran.r-project.org/package=cocor>. A web front-end to conveniently access the functionality of the cocor package is available at <http://comparingcorrelations.org>.

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Introduction

Determining the relationship between two variables is at the heart of many research endeavours. In the social sciences, the most popular statistical method to quantify the magnitude of an association between two numeric variables is the Pearson product-moment correlation. It indicates the strength of a linear relationship between two variables, which may be either positive, negative, or zero. In many research contexts, it is necessary to compare the magnitude of two such correlations, for example, if a researcher wants to know whether an association changed after a treatment, or whether it differs between two groups of interest. When comparing correlations, a test of significance is necessary to control for the possibility of an observed difference occurring simply by chance. However, many introductory statistics textbooks [1–5] do not even mention significance tests for correlations. Also in research practice, the necessity of conducting a proper statistical test when comparing the magnitude of correlations is often ignored. For example, in neuroscientific investigations, correlations between behavioral measures and brain areas are often determined to identify the brain area that is most strongly involved in a given task. Rousselet and Pernet [6] criticized that such studies rarely provide quantitative tests of the difference between correlations. Instead, many authors fall prey to a statistical fallacy, and wrongly consider the existence of a significant and a nonsignificant correlation as providing sufficient evidence for a significant difference between these two

correlations. Nieuwenhuis, Forstmann, and Wagenmakers [7] also found that, when making a comparison between correlations, researchers frequently interpreted a significant correlation in one condition and a nonsignificant correlation in another condition as providing evidence for different correlations in the two conditions. Such an interpretation, however, is fallacious. As pointed out by Rosnow and Rosenthal [8], “God loves the .06 nearly as much as the .05”. To make a valid, meaningful, and interpretable comparison between two correlations, it is necessary to directly contrast the two correlations under investigation using an appropriate statistical test [7].

Even when recognizing the importance of a formal statistical test of the difference between correlations, the researcher has many different significance tests to choose from, and the choice of the correct method is vital. Before picking a test, researchers have to distinguish between the following three cases: (1) The correlations were measured in two independent groups A and B. This case applies, for example, if a researcher wants to compare the correlations between anxiety and extraversion in two different groups A and B ($\rho_A = \rho_B$). If the two groups are dependent, the relationship between them needs further differentiation: (2) The two correlations can be overlapping ($\rho_{A12} = \rho_{A23}$), i.e., the correlations have one variable in common. ρ_{A12} and ρ_{A23} refer to the population correlations in group A between variables 1 and 2 and variables 2 and 3, respectively. For instance, a researcher may be interested in determining whether the correlation between anxiety and extraversion is smaller than between anxiety and diligence within the same group A. (3) In the case of two dependent correlations, the two correlations can also be nonoverlapping ($\rho_{A12} = \rho_{A34}$), i.e., they have no variable in common. This case applies, for example, if a researcher wants to determine whether the correlation between anxiety and extraversion is higher than the correlation between intelligence and creativity within the same group. A researcher also faces nonoverlapping dependent correlations when investigating whether the correlation between two variables is higher before rather than after a treatment provided to the same group.

For each of these three cases, various tests have been proposed. An overview of the tests for comparing independent correlations is provided in Table 1, and for comparing dependent correlations—overlapping and nonoverlapping—in Tables 2 and 3, respectively. May and Hittner [9] compared the statistical power and Type I error rate of several tests for dependent overlapping correlations, and found no test to be uniformly preferable. Instead, they concluded that the best choice is influenced by sample size, predictor intercorrelation, effect size, and predictor-criterion correlation. Because no clear recommendation for any of these tests can be formulated that applies under all circumstances, and because different methods may be optimal for a research question at hand, it is important that researchers are provided with a tool that allows them to choose freely between all available options. Detailed discussions of the competing tests for comparing dependent overlapping correlations are given in Dunn and Clark [10], Hittner, May, and Silver [11], May and Hittner [9], Neill and Dunn [12], and Steiger [13]. For the case of dependent nonoverlapping correlations, the pros and cons of various tests are discussed in Raghunathan, Rosenthal, and Rubin [14], Silver, Hittner, and May [15], and Steiger [13]. In contrast to most other approaches, Zou [16] has advocated a test that is based on the

Table 1. Software implementing tests for comparing two correlations based on independent groups.

Test	psych	multilevel	Weaver & Wuensch	cocor
Fisher's [20] z	•	•	•	•
Zou's [16] confidence interval			•	•

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Table 2. Software implementing tests for comparing two correlations based on dependent groups with overlapping variables.

Test	psych	multilevel	DEPCORR	DEPCOR	Weaver & Wuensch	cocor
Pearson and Filon's [21] z						•
Hotelling's [22] t			•			•
Williams' [23] t	•	•	•	•	•	•
Olkin's [24] z			•			•
Dunn and Clark's [25] z			•	•		•
Hendrickson et al.'s [26] modification of Williams' [23] t			•			•
Steiger's [13] modification of Dunn and Clark's [25] z			•	•		•
Meng, Rosenthal, and Rubin's [27] z			•	•		•
Hittner et al.'s [11] modification of Dunn and Clark's [25] z				•		•
Zou's [16] confidence interval					•	•

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computation of confidence intervals, which are often regarded as superior to significance testing because they separately indicate the magnitude and the precision of an estimated effect [17, 18]. Confidence intervals can be used to test whether a correlation significantly differs from zero or from some constant, and whether the difference between two correlations exceeds a predefined threshold. Zou's confidence interval [16] is available for comparisons of independent and dependent correlations with either overlapping or nonoverlapping variables. The tests proposed by Zou [16] have been compared to other confidence interval procedures by Wilcox [19].

Existing Software

Many popular statistics programs do not provide any, or only a subset of the significance tests described above. Moreover, existing programs that allow for statistical comparisons between correlations are isolated stand-alone applications and do not come with a graphical user interface (GUI). For example, DEPCOR [28] is a program that is limited to comparisons of two dependent correlations—either overlapping or nonoverlapping. The program is written in Fortran and runs in a DOS command prompt console under the Windows platform. Another available package, DEPCORR [29], is an SAS macro [30] for comparing two dependent overlapping correlations. The latest release of SAS/STAT software (version 9.4) runs on Windows and Linux systems. However, DEPCORR has no GUI and covers only one of the three cases described above. The two packages *psych* [31] and *multilevel* [32] for the R programming language [33] also offer functions to compare two dependent or independent correlations. However, each of these functions covers only one or two of the many different available tests of comparison, and there is no GUI available to access the functions of the packages. Weaver and

Table 3. Software implementing tests for comparing two correlations based on dependent groups with nonoverlapping variables.

Test	psych	DEPCORR	Weaver & Wuensch	cocor
Pearson and Filon's [21] z			•	•
Dunn and Clark's [25] z	•	•		•
Steiger's [13] modification of Dunn and Clark's [25] z	•	•		•
Raghunathan, Rosenthal, and Rubin's [14] modification of Pearson and Filon's [21] z			•	•
Silver, Hittner, and May's [15] modification of Dunn and Clark's [25] z		•		•
Zou's [16] confidence interval			•	•

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Wuensch [34] recently published thoroughly documented scripts for comparing dependent or independent correlations in SPSS and SAS.

cocor

With `cocor` (version 1.1-0), we provide a comprehensive solution to compare two correlations based on either dependent or independent groups. The `cocor` package enhances the R programming environment [33], which is freely available for Windows, Mac, and Linux systems and can be downloaded from CRAN (<http://cran.r-project.org/package=cocor>). All that is needed to install the `cocor` package is to type `install.packages("cocor")` in the R console, and the functionality of the package is made available by typing `library("cocor")`. The function `cocor()` calculates and compares correlations from raw data. The underlying variables are specified via a formula interface (see Fig. 1). If raw data are not available, `cocor` offers three functions to compare correlation coefficients that have already

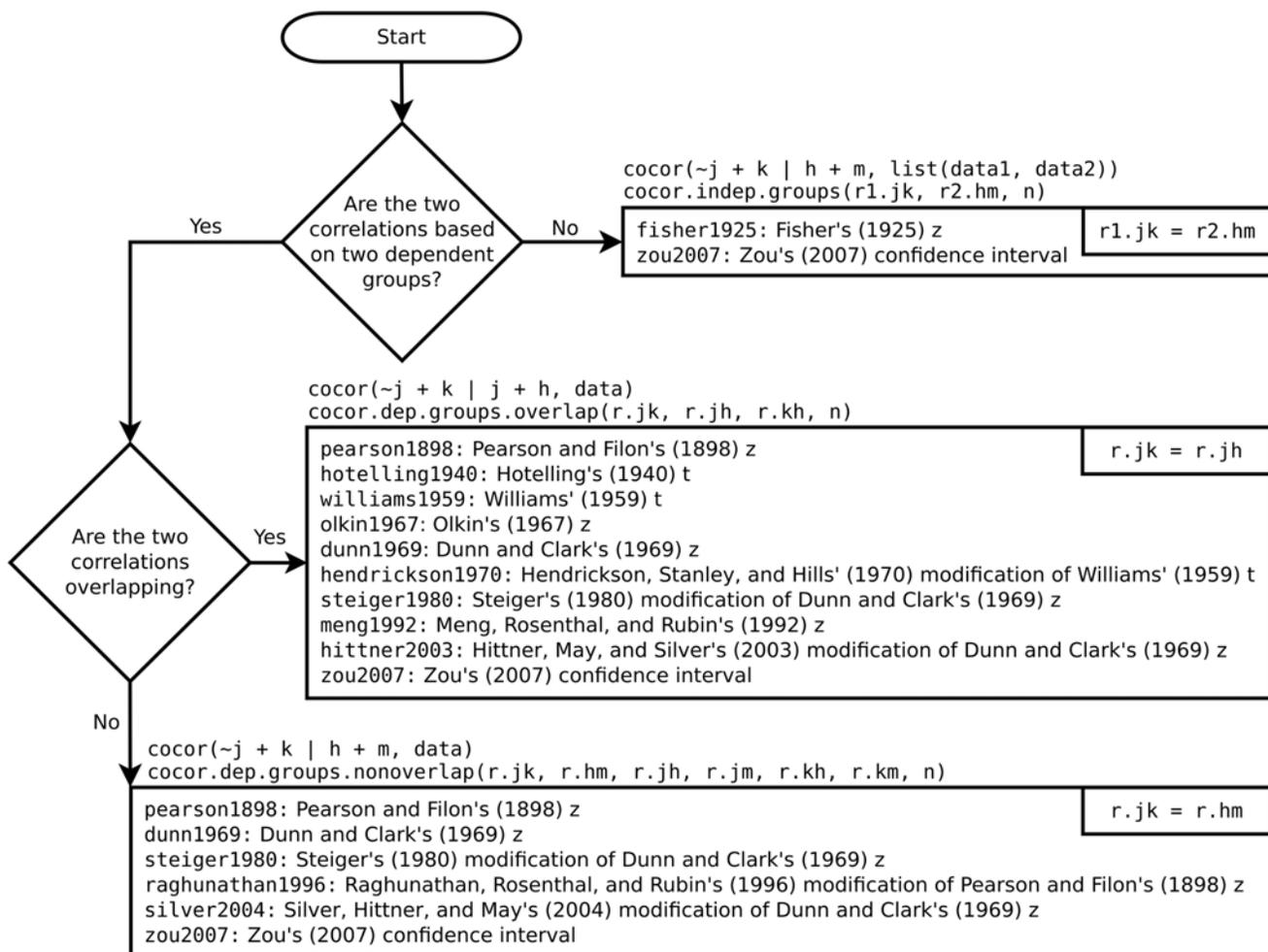


Fig 1. A flowchart of how to use the four main functions of cocor, displaying all available tests. For each case, an example of the formula passed as an argument to the `cocor()` function and the required correlation coefficients for the functions `cocor.indep.groups()`, `cocor.dep.groups.overlap()`, and `cocor.dep.groups.nonoverlap()` are given. The test label before the colon may be passed as a function argument to calculate specific tests only.

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been determined. The function `cocor.indep.groups()` compares two independent correlations, whereas the functions `cocor.dep.groups.overlap()` and `cocor.dep.groups.nonoverlap()` compare two dependent overlapping or nonoverlapping correlations, respectively. Internally, `cocor()` passes the calculated correlations coefficients to one of these three functions. All functions allow to specify the argument `null.value` to test whether the difference between the correlations exceeds a given threshold using the confidence intervals by Zou [16]. The results are either returned as an S4 object of class `cocor` whose input and result parameters can be obtained using the `get.cocor.input()` and `get.cocor.results()` functions, respectively. Optionally, results may also be returned as a list of class `htest`. By default, all tests available are calculated. Specific tests can be selected by passing a test label to the function using the `test` argument. The flowchart in Fig. 1 shows how to access the available tests and lists them with their individual test label (e.g., `zou2007`). The formulae of all implemented tests are detailed in S1 Appendix.

A comparison of `cocor` with competing software can be found in Tables 1–3. These tables show that `cocor` offers a larger variety of tests and a more comprehensive approach than all previous solutions. In particular, `cocor` is the first R package to implement the tests by Zou [16]. Further unique features of the `cocor` package are the formula interface for comparing correlations that extracts the correlations from data, and the unified function for statistical tests capable of comparing both, independent and dependent correlations with either overlapping or nonoverlapping variables.

Some limitations of `cocor` should be acknowledged, however. First, `cocor` is limited to the comparison of two correlations. The simultaneous comparison of more than two correlations needs tests that go beyond the scope of the present contribution [35–37]. Second, `cocor` does not allow one to employ structural equation models that are needed for more advanced, but also more complex approaches to the statistical comparison of correlations [38, 39].

GUIs for cocor

There are two convenient ways to use `cocor` via a GUI. First, the package includes a plugin for the platform independent R front-end Rkward [40] (Fig. 2). Second, for those unfamiliar with R, a web interface is also available at <http://comparingcorrelations.org> (Fig. 3).

Thus, `cocor` offers the best of two worlds: On the one hand, it has the power of a scripting language with the possibility of automation. On the other hand, the two available GUIs allow even inexperienced users to use `cocor` in a convenient way. As `cocor` is embedded in the R environment for statistical computing, it allows for a seamless integration into R analyses. R code can be generated via the GUIs and used for subsequent batch analyses. Since `cocor` is published under the GNU General Public License (GPL; version 3 or higher), all users are invited to inspect, use, copy, modify, and redistribute the code under the same license.

Code Examples

In the following, using fictional data, examples are given for all three cases that may occur when comparing correlations.

Comparison of Two Correlations Based on Independent Groups

The first example presents code for the comparison of the correlations between a score achieved on a logic task (`logic`) and an intelligence measure A (`intelligence.a`) in two different groups. Note that the underlying data set (`aptitude`) is a list that contains two separate data sets.

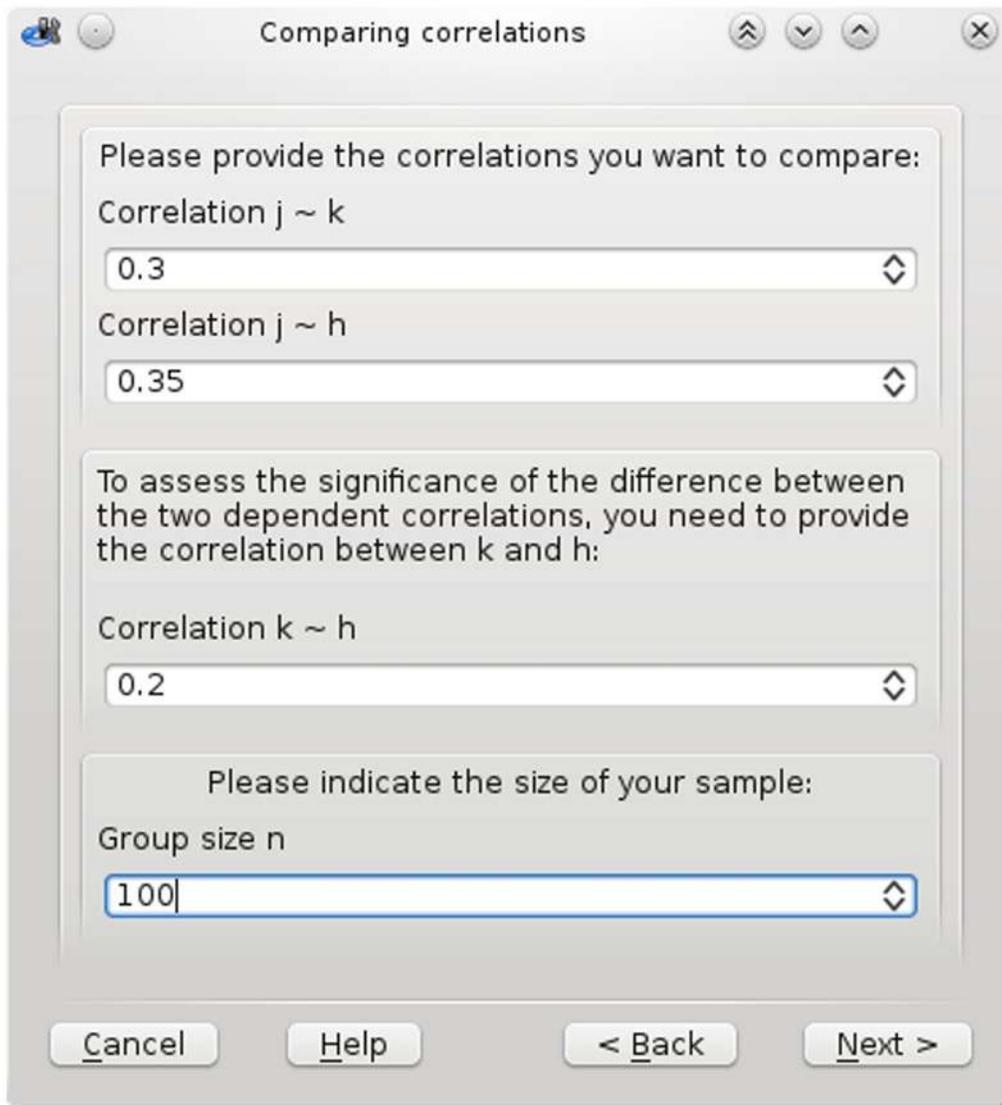


Fig 2. Screenshot of the cocor GUI plugin for RKWard.

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```
R> require ("cocor")
R> data ("aptitude")
R> cocor (~logic+intelligence.a | logic+intelligence.a,
+ aptitude)
  Results of a comparison of two correlations based on independent
  groups
Comparison between r1.jk (logic, intelligence.a) = 0.3213 and r2.
hm (logic, intelligence.a) = 0.2024
Difference: r1.jk-r2.hm = 0.1189
Data: sample1: j = logic, k = intelligence.a; sample2: h = logic,
m = intelligence.a
Group sizes: n1 = 291, n2 = 334
Null hypothesis: r1.jk is equal to r2.hm
```

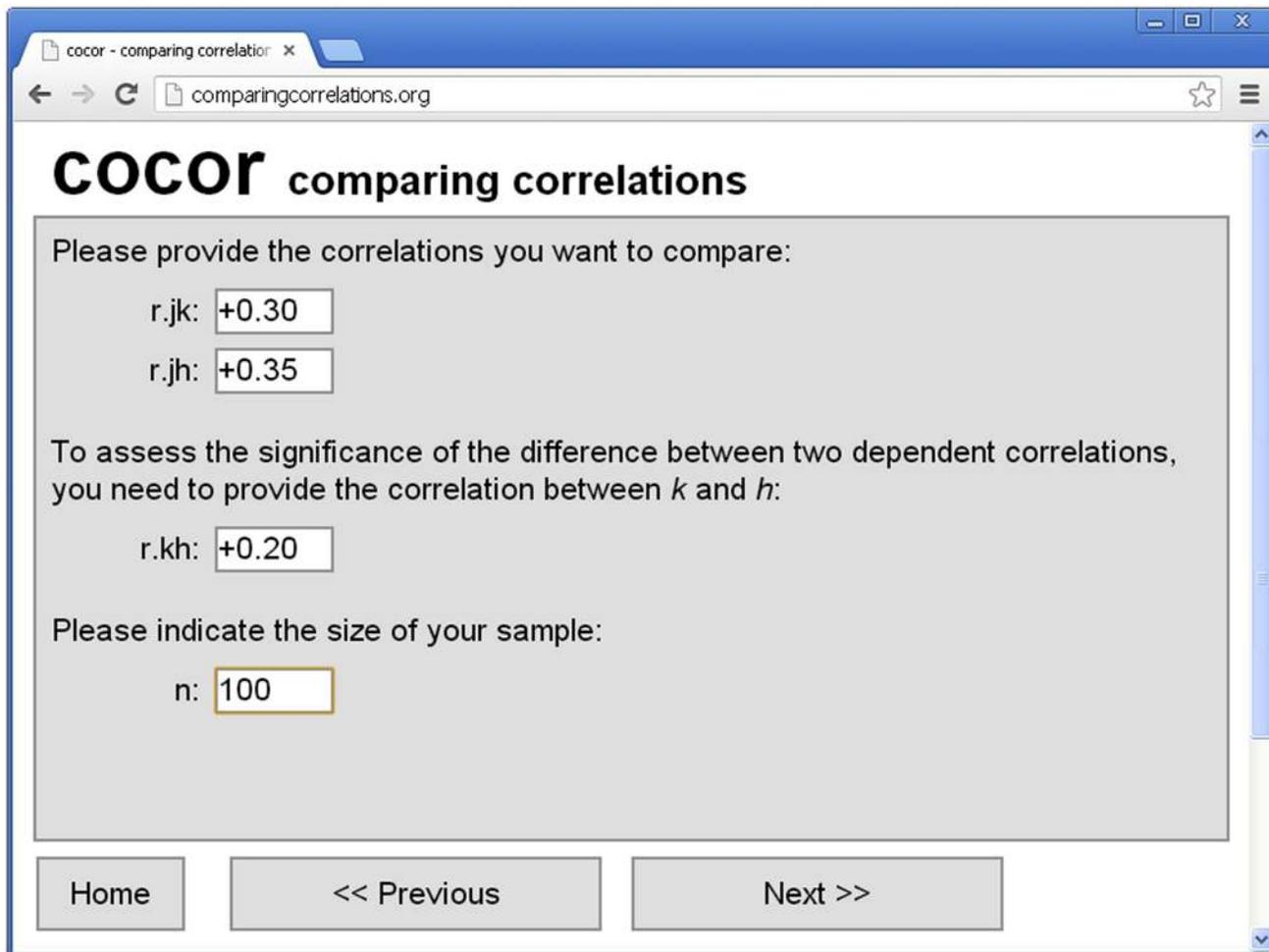


Fig 3. Screenshot of the cocor web interface on <http://comparingcorrelations.org>.

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```

Alternative hypothesis: r1.jk is not equal to r2.hm (two-sided)
Alpha: 0.05
fisher1925: Fisher's z (1925)
  z = 1.5869, p-value = 0.1125
  Null hypothesis retained
zou2007: Zou's (2007) confidence interval
  95% confidence interval for r1.jk-r2.hm: -0.0281 0.2637
  Null hypothesis retained (Interval includes 0)
  
```

In this example, the test result indicates that the difference between the two correlations $r1.jk$ and $r2.hm$ is not significant, and the null hypothesis cannot be rejected. Alternatively, the same comparison can also be conducted based on the correlation coefficients and the group sizes using the function `cocor.indep.groups()`.

```

R> cocor.indep.groups (r1.jk = 0.3213, r2.hm = 0.2024, n1 = 291,
+ n2 = 334)
  
```

Comparison of Two Overlapping Correlations Based on Dependent Groups

The second example code determines whether the correlation between a score achieved on general knowledge questions (`knowledge`) and an intelligence measure A (`intelligence.a`) differs from the correlation between a score achieved on a logic task (`logic`) and the same intelligence measure A (`intelligence.a`) within a group of $n = 291$ persons.

```
R> cocor (~knowledge + intelligence.a | logic ++ intelligence.a,
  aptitude[[ "sample1" ]])
  Results of a comparison of two overlapping correlations based on
  dependent groups
  Comparison between r.jk (intelligence.a, knowledge) = 0.1038 and
  r.jh (intelligence.a, logic) = 0.3213
  Difference: r.jk-r.jh = -0.2175
  Related correlation: r.kh = 0.0257
  Data: aptitude[[ "sample1" ]]: j = intelligence.a, k = knowledge,
  h = logic
  Group size: n = 291
  Null hypothesis: r.jk is equal to r.jh
  Alternative hypothesis: r.jk is not equal to r.jh (two-sided)
  Alpha: 0.05
  pearson1898: Pearson and Filon' s z (1898)
    z = -2.7914, p-value = 0.0052
    Null hypothesis rejected
  hotelling1940: Hotelling' s t (1940)
    t = -2.8066, df = 288, p-value = 0.0053
    Null hypothesis rejected
  williams1959: Williams' t (1959)
    t = -2.7743, df = 288, p-value = 0.0059
    Null hypothesis rejected
  olkin1967: Olkin' s z (1967)
    z = -2.7914, p-value = 0.0052
    Null hypothesis rejected
  dunn1969: Dunn and Clark' s z (1969)
    z = -2.7595, p-value = 0.0058
    Null hypothesis rejected
  hendrickson1970: Hendrickson, Stanley, and Hills' (1970) modifi-
  cation of Williams' t (1959)
    t = -2.8065, df = 288, p-value = 0.0053
    Null hypothesis rejected
  steiger1980: Steiger' s (1980) modification of Dunn and Clark' s z
  (1969) using average correlations
    z = -2.7513, p-value = 0.0059
    Null hypothesis rejected
  meng1992: Meng, Rosenthal, and Rubin' s z (1992)
    z = -2.7432, p-value = 0.0061
    Null hypothesis rejected
  95% confidence interval for r.jk-r.jh: -0.3925 -0.0654
```

```

Null hypothesis rejected (Interval does not include 0)
hittner2003: Hittner, May, and Silver's (2003) modification of
Dunn and Clark's z (1969) using a backtransformed average Fisher's
(1921) Z procedure
z = -2.7505, p-value = 0.0059
Null hypothesis rejected
zou2007: Zou's (2007) confidence interval
95% confidence interval for r.jk-r.jh: -0.3689 -0.0630
Null hypothesis rejected (Interval does not include 0)

```

The results of all tests lead to the convergent conclusion that the difference between the two correlations $r.jk$ and $r.jh$ is significant, and the null hypothesis should be rejected. Alternatively, the same comparison can also be conducted based on the correlation coefficients and the group size using the function `cocor.dep.groups.overlap()`.

```

R> cocor.dep.groups.overlap (r.jk=0.1038, r.jh=0.3213, + r.
kh=0.0257, n=291)

```

Comparison of Two Nonoverlapping Correlations Based on Dependent Groups

The third example code tests whether the correlation between a score achieved on general knowledge questions (`knowledge`) and an intelligence measure A (`intelligence.a`) differs from the correlation between a score achieved on a logic task (`logic`) and an intelligence measure B (`intelligence.b`) within the same group of $n = 291$ persons.

```

R> cocor (~knowledge + intelligence.a | logic ++ intelligence.b,
apitude[["sample1"]])
Results of a comparison of two nonoverlapping correlations based
on dependent groups
Comparison between r.jk (knowledge, intelligence.a) = 0.1038 and
r.hm (logic, intelligence.b) = 0.2679
Difference: r.jk-r.hm = -0.164
Related correlations: r.jh=0.0257, r.jm=0.1713, r.kh=0.3213,
r.km=0.4731
Data: apitude[["sample1"]]: j = knowledge, k = intelligence.a,
h = logic, m = intelligence.b
Group size: n = 291
Null hypothesis: r.jk is equal to r.hm
Alternative hypothesis: r.jk is not equal to r.hm (two-sided)
Alpha: 0.05
pearson1898: Pearson and Filon's z (1898)
z = -2.0998, p-value = 0.0357
Null hypothesis rejected
dunn1969: Dunn and Clark's z (1969)
z = -2.0811, p-value = 0.0374
Null hypothesis rejected
steiger1980: Steiger's (1980) modification of Dunn and Clark's z
(1969) using average correlations

```

```
z = -2.0755, p-value = 0.0379
Null hypothesis rejected
raghunathan1996: Raghunathan, Rosenthal, and Rubin's (1996) modification of Pearson and Filon's z (1898)
z = -2.0811, p-value = 0.0374
Null hypothesis rejected
silver2004: Silver, Hittner, and May's (2004) modification of Dunn and Clark's z (1969) using a backtransformed average Fisher's (1921) Z procedure
z = -2.0753, p-value = 0.0380
Null hypothesis rejected
zou2007: Zou's (2007) confidence interval
95% confidence interval for r.jk-r.hm: -0.3162 -0.0095
Null hypothesis rejected (Interval does not include 0)
```

Also in this example, the test results converge in showing that the difference between the two correlations $r.jk$ and $r.hm$ is significant, and the null hypothesis should be rejected. Alternatively, the same comparison can also be conducted based on the correlation coefficients and the group size using the function `cocor.dep.groups.nonoverlap()`.

```
R> cocor.dep.groups.nonoverlap(r.jk=0.1038, r.hm=0.2679, + r.jh=0.0257, r.jm=0.1713, r.kh=0.3213, + r.km=0.4731, n=291)
```

Discussion and Summary

In this article, we introduced `cocor`, a free software package for the R programming language [33]. The `cocor` package provides a wide range of tests for comparisons of independent and dependent correlations with either overlapping or nonoverlapping variables. Unlike existing solutions, `cocor` is available for scripting within the R environment, while offering two convenient GUIs: a plugin for RKWard [40] and a web interface. Thus, `cocor` enables users of all knowledge levels to access a large variety of tests for comparing correlations in a convenient and user-friendly way.

Supporting Information

S1 Appendix. Documentation of All Tests Implemented in cocor.
(PDF)

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Author Contributions

Wrote the paper: BD JM. Software development: BD.

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CORRECTION

Correction: cocor: A Comprehensive Solution for the Statistical Comparison of Correlations

The PLOS ONE Staff

The URL in the Data Availability statement for this paper is incorrect. The correct statement is: “Data Availability Statement: The cocor R package can be downloaded from <http://cran.r-project.org/package=cocor>. A web front-end to conveniently access the functionality of the cocor package is available at <http://comparingcorrelations.org>.” The publisher apologizes for the error.

There is an error in the URL in the second sentence of the subsection “cocor” in the Introduction. The correct sentence should be: “The cocor package enhances the R programming environment [33], which is freely available for Windows, Mac, and Linux systems and can be downloaded from CRAN (<http://cran.r-project.org/package=cocor>).” The publisher apologizes for the error.

There is an error in the URL in reference 31 of the References. The correct reference should be: “Revelle W. psych: Procedures for psychological, psychometric, and personality research; 2014. R package version 1.4.8. Available: <http://cran.R-project.org/package=psych>. Accessed 21 February 2015.” The publisher apologizes for the error.

There is an error in the URL in reference 32 of the References. The correct reference should be: “Bliese P. multilevel: Multilevel Functions; 2013. R package version 2.5. Available: <http://cran.R-project.org/package=multilevel>. Accessed 21 February 2015.” The publisher apologizes for the error.

There are several errors in the “Comparison of Two Correlations Based on Independent Groups” subsection of the Code Examples section. The publisher apologizes for the error. Please view the correct code here. Figs [4](#) and [5](#)



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```
R> requires("cocor")
R> data("aptitude")
R> cocor(~logic + intelligence.a | logic + intelligence.a,
+ aptitude)
```

Results of a comparison of two correlations based on independent groups

```
Comparison between r1.jk (logic, intelligence.a) = 0.3213
and r2.hm (logic, intelligence.a) = 0.2024
Difference: r1.jk - r2.hm = 0.1189
Data: sample1: j = logic, k = intelligence.a; sample2:
h = logic, m = intelligence.a
Group sizes: n1 = 291, n2 = 334
Null hypothesis: r1.jk is equal to r2.hm
Alternative hypothesis: r1.jk is not equal to r2.hm
(two-sided)
Alpha: 0.05

fisher1925: Fisher's z (1925)
z = 1.5869, p-value = 0.1125
Null hypothesis retained

zou2007: Zou's (2007) confidence interval
95% confidence interval for r1.jk - r2.hm: -0.0281 0.2637
Null hypothesis retained (Interval includes 0)
```

Fig 4.

doi:10.1371/journal.pone.0131499.g001

```
R> cocor.indep.groups(r1.jk=0.3213, r2.hm=0.2024, n1=291,
+ n2=334)
```

Fig 5.

doi:10.1371/journal.pone.0131499.g002

There are several errors in the “Comparison of Two Overlapping Correlation Based on Dependent Groups” subsection of the Code Examples section. The publisher apologizes for the error. Please view the correct code here. Figs [6](#) and [7](#)

```

R> cocor(~knowledge + intelligence.a | logic +
+ intelligence.a, aptitude[["sample1"]])

Results of a comparison of two overlapping correlations
based on dependent groups

Comparison between r.jk (intelligence.a, knowledge) = 0.1038
and r.jh (intelligence.a, logic) = 0.3213
Difference: r.jk - r.jh = -0.2175
Related correlation: r.kh = 0.0257
Data: aptitude[["sample1"]]: j = intelligence.a,
k = knowledge, h = logic
Group size: n = 291
Null hypothesis: r.jk is equal to r.jh
Alternative hypothesis: r.jk is not equal to r.jh (two-sided)
Alpha: 0.05

pearson1898: Pearson and Filon's z (1898)
z = -2.7914, p-value = 0.0052
Null hypothesis rejected

hotelling1940: Hotelling's t (1940)
t = -2.8066, df = 288, p-value = 0.0053
Null hypothesis rejected

williams1959: Williams' t (1959)
t = -2.7743, df = 288, p-value = 0.0059
Null hypothesis rejected

olkin1967: Olkin's z (1967)
z = -2.7914, p-value = 0.0052
Null hypothesis rejected

dunn1969: Dunn and Clark's z (1969)
z = -2.7595, p-value = 0.0058
Null hypothesis rejected

hendrickson1970: Hendrickson, Stanley, and Hills' (1970)
modification of Williams' t (1959)
t = -2.8065, df = 288, p-value = 0.0053
Null hypothesis rejected

steiger1980: Steiger's (1980) modification of Dunn and
Clark's z (1969) using average correlations
z = -2.7513, p-value = 0.0059
Null hypothesis rejected

meng1992: Meng, Rosenthal, and Rubin's z (1992)
z = -2.7432, p-value = 0.0061
Null hypothesis rejected
95% confidence interval for r.jk - r.jh: -0.3925 -0.0654
Null hypothesis rejected (Interval does not include 0)

hittner2003: Hittner, May, and Silver's (2003) modification
of Dunn and Clark's z (1969) using a backtransformed average
Fisher's (1921) Z procedure
z = -2.7505, p-value = 0.0059
Null hypothesis rejected

zou2007: Zou's (2007) confidence interval
95% confidence interval for r.jk - r.jh: -0.3689 -0.0630
Null hypothesis rejected (Interval does not include 0)

```

Fig 6.

doi:10.1371/journal.pone.0131499.g003

```
R> cocor.dep.groups.overlap(r.jk=0.1038, r.jh=0.3213,  
+ r.kh=0.0257, n=291)
```

Fig 7.

doi:10.1371/journal.pone.0131499.g004

There are several errors in the “Comparison of Two Nonoverlapping Correlations Based on Dependent Groups” subsection of the Code Examples section. The publisher apologizes for the error. Please view the correct code here. Figs [8](#) and [9](#)

```
R> cocor("knowledge + intelligence.a | logic +
+ intelligence.b, aptitude[["sample1"]]")

Results of a comparison of two nonoverlapping correlations
based on dependent groups

Comparison between r.jk (knowledge, intelligence.a) = 0.1038
and r.hm (logic, intelligence.b) = 0.2679
Difference: r.jk - r.hm = -0.164
Related correlations: r.jh = 0.0257, r.jm = 0.1713,
r.kh = 0.3213, r.km = 0.4731
Data: aptitude[["sample1"]]: j = knowledge,
k = intelligence.a, h = logic, m = intelligence.b
Group size: n = 291
Null hypothesis: r.jk is equal to r.hm
Alternative hypothesis: r.jk is not equal to r.hm (two-sided)
Alpha: 0.05

pearson1898: Pearson and Filon's z (1898)
z = -2.0898, p-value = 0.0357
Null hypothesis rejected

dunn1969: Dunn and Clark's z (1969)
z = -2.0811, p-value = 0.0374
Null hypothesis rejected

steiger1980: Steiger's (1980) modification of Dunn and
Clark's z (1969) using average correlations
z = -2.0765, p-value = 0.0379
Null hypothesis rejected

raghunathan1996: Raghunathan, Rosenthal, and Rubin's (1996)
modification of Pearson and Filon's z (1898)
z = -2.0811, p-value = 0.0374
Null hypothesis rejected

silver2004: Silver, Hittner, and May's (2004) modification
of Dunn and Clark's z (1969) using a backtransformed average
Fisher's (1921) Z procedure
z = -2.0753, p-value = 0.0380
Null hypothesis rejected

zou2007: Zou's (2007) confidence interval
95% confidence interval for r.jk - r.hm: -0.3162 -0.0085
Null hypothesis rejected (Interval does not include 0)
```

Fig 8.

doi:10.1371/journal.pone.0131499.g005

```
R> cocor.dep.groups.nonoverlap(r.jk = 0.1038, r.hm = 0.2679,
+ r.jh = 0.0257, r.jm = 0.1713, r.kh = 0.3213,
+ r.km = 0.4731, n=291)
```

Fig 9.

doi:10.1371/journal.pone.0131499.g006

In the Supporting Information file [S1 Appendix](#), there are errors in Equations 4, 32, and 51. These equations should contain a “+” sign before the square root sign instead of a “-” sign. The publisher apologizes for the error. Please view the correct [S1 Appendix](#) below.

Supporting Information

S1 Appendix. Documentation of All Tests Implemented in cocor.
(PDF)

Reference

1. Diedenhofen B, Musch J (2015) cocor: A Comprehensive Solution for the Statistical Comparison of Correlations. PLoS ONE 10(4): e0121945. doi: [10.1371/journal.pone.0121945](https://doi.org/10.1371/journal.pone.0121945)

S1 Appendix. Documentation of All Tests Implemented in cocor

This Appendix is part of the article *cocor: A Comprehensive Solution for the Statistical Comparison of Correlations* by Birk Diedenhofen¹ and Jochen Musch published in PLOS ONE. In the following, the formulae of all tests implemented in the R package [1] *cocor* (version 1.1-0) are provided. z statistics are based on a normal distribution, whereas t statistics rely on a Student's t -distribution with given degrees of freedom. Some tests make use of Fisher's [2, p 26] r -to- Z transformation:

$$Z = \frac{1}{2}(\ln(1+r) - \ln(1-r)). \quad (1)$$

Tests for Comparison of Two Correlations Based on Independent Groups

The function `cocor.indep.groups()` implements tests for the comparison of two correlations based on independent groups.

fisher1925: Fisher's [3] z

This significance test was first described by Fisher [3, pp 161–168] and its test statistic z is calculated as

$$z = \frac{Z_1 - Z_2}{\sqrt{\frac{1}{n_1-3} + \frac{1}{n_2-3}}}. \quad (2)$$

Z_1 and Z_2 are the two Z transformed correlations that are being compared. n_1 and n_2 specify the size of the two groups the correlations are based on. Equation 2 is also given for example in Peters and van Voorhis [4, p 188] and Cohen, Cohen, West, and Aiken [5, p 49, formula 2.8.11].

zou2007: Zou's [6] confidence interval

This test calculates the confidence interval of the difference between the two correlation coefficients r_1 and r_2 . If the confidence interval includes zero, the null hypothesis that the two correlations are equal must be retained. If the confidence interval does not include zero, the null hypothesis has to be rejected. A lower and upper bound for the interval (L and U , respectively) is given by

$$L = r_1 - r_2 - \sqrt{(r_1 - l_1)^2 + (u_2 - r_2)^2} \quad (3)$$

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and

$$U = r_1 - r_2 + \sqrt{(u_1 - r_1)^2 + (r_2 - l_2)^2} \quad (4)$$

[6, p 409]. A lower and upper bound for the confidence interval of r_1 (l_1 and u_1) and r_2 (l_2 and u_2) are calculated as

$$l = \frac{\exp(2l') - 1}{\exp(2l') + 1}, \quad (5)$$

$$u = \frac{\exp(2u') - 1}{\exp(2u') + 1} \quad (6)$$

[6, p 406], where

$$l', u' = Z \pm z_{\frac{\alpha}{2}} \sqrt{\frac{1}{n-3}} \quad (7)$$

[6, p 406]. α denotes the desired alpha level of the confidence interval, whereas n specifies the size of the group the correlation is based on.

Tests for Comparison of Two Overlapping Correlations Based on Dependent Groups

The function `cocor.dep.groups.overlap()` implements tests for the comparison of two overlapping correlations based on dependent groups. In the following, r_{jk} and r_{jh} are the two correlations that are being compared; Z_{jk} and Z_{jh} are their Z transformed equivalents. r_{kh} is the related correlation that is additionally required. n specifies the size of the group the two correlations are based on.

pearson1898: Pearson and Filon's [7] z

This test was proposed by Pearson and Filon [7, p 259, formula xxxvii]. The test statistic z is computed as

$$z = \frac{\sqrt{n}(r_{jk} - r_{jh})}{\sqrt{(1 - r_{jk}^2)^2 + (1 - r_{jh}^2)^2 - 2k}} \quad (8)$$

[8, p 246, formula 4], where

$$k = r_{kh}(1 - r_{jk}^2 - r_{jh}^2) - \frac{1}{2}(r_{jk}r_{jh})(1 - r_{jk}^2 - r_{jh}^2 - r_{kh}^2) \quad (9)$$

[8, p 245, formula 3].

hotelling1940: Hotelling's [9] t

The test statistic t is given by

$$t = \frac{(r_{jk} - r_{jh})\sqrt{(n-3)(1+r_{kh})}}{\sqrt{2|R|}} \quad (10)$$

[9, p 278, formula 7] with $df = n - 3$, where

$$|R| = 1 + 2r_{jk}r_{jh}r_{kh} - r_{jk}^2 - r_{jh}^2 - r_{kh}^2 \quad (11)$$

[9, p 278]. Equation 10 is also given in Steiger [8, p 246], Glass and Stanley [10, p 311, formula 15.7], and Hittner et al. [11, p 152].

williams1959: Williams' [12] t

This test is a modification of Hotelling's [9] t and was suggested by Williams [12]. Two mathematically different formulae for Williams' t can be found in the literature [11, p 152]. This is the version that Hittner et al. [11, p 152] labeled as "standard Williams' t ":

$$t = (r_{jk} - r_{jh})\sqrt{\frac{(n-1)(1+r_{kh})}{2\left(\frac{n-1}{n-3}\right)|R| + \bar{r}^2(1-r_{kh})^3}} \quad (12)$$

with $df = n - 3$, where

$$\bar{r} = \frac{r_{jk} + r_{jh}}{2} \quad (13)$$

and

$$|R| = 1 + 2r_{jk}r_{jh}r_{kh} - r_{jk}^2 - r_{jh}^2 - r_{kh}^2. \quad (14)$$

An alternative formula for Williams' t – termed as "Williams' modified t per Hendrickson, Stanley, and Hills" [13] by Hittner et al. [11, p 152] – is implemented in `cocor` as `hendrickson1970` (see Equation 18 below). Equation 12 is also given in Steiger [8, p 246, formula 7] and Neill and Dunn [14, p 533].

Results from Equation 12 are in accordance with the results of DEPCORR [15] and DEPCOR [16]. However, we found several typographical errors in formulae that also claim to compute Williams' t . For example, the formula reported by Boyer, Palachek, and Schucany [17, p 76] contains an error because

the term $(1 - r_{rk})$ is not being cubed. There are also typographical errors in the formula described by Hittner et al. [11, p 152]. For example, $r_{jk} - r_{jh}$ is divided instead of being multiplied by the square root term, and in the denominator of the fraction in the square root term, there are additional parentheses so that the whole denominator is multiplied by 2. These same errors can also be found in Wilcox and Tian [18, p 107, formula 1].

olkin1967: Olkin's [19] z

In the original article by Olkin [19, p 112] and in Hendrickson et al. [13, p 190, formula 2], the reported formula contains a typographical error. Hendrickson and Collins [20, p 639] provide a corrected version. In the revised version, however, n in the numerator is decreased by 1. The `cocor` package implements the corrected formula without the decrement. The formula implemented in `cocor` is used by Glass and Stanley [21, p 313, formula 14.19], Hittner et al. [11, p 152], and May and Hittner [22, p 259] [23, p 480]:

$$z = \frac{(r_{jk} - r_{jh})\sqrt{n}}{\sqrt{(1 - r_{jk}^2)^2 + (1 - r_{jh}^2)^2 - 2r_{kh}^3 - (2r_{kh} - r_{jk}r_{jh})(1 - r_{kh}^2 - r_{jk}^2 - r_{jh}^2)}}. \quad (15)$$

dunn1969: Dunn and Clark's [24] z

The test statistic z of this test is calculated as

$$z = \frac{(Z_{jk} - Z_{jh})\sqrt{n - 3}}{\sqrt{2 - 2c}} \quad (16)$$

[24, p 370, formula 15], where

$$c = \frac{r_{kh}(1 - r_{jk}^2 - r_{jh}^2) - \frac{1}{2}r_{jk}r_{jh}(1 - r_{jk}^2 - r_{jh}^2 - r_{kh}^2)}{(1 - r_{jk}^2)(1 - r_{jh}^2)} \quad (17)$$

[24, p 368, formula 8].

hendrickson1970: Hendrickson, Stanley, and Hills [13] modification of Williams' [12] t

This test is a modification of Hotelling's [9] t and was suggested by Williams [12]. Two mathematically different formulae of Williams' [12] t can be found in the literature. `hendrickson1970` is the version that Hittner et al. [11, p 152] labeled as "Williams' modified t per Hendrickson, Stanley, and Hills" [13].

An alternative formula termed as "standard Williams' t " by Hittner et al. [11, p 152] is implemented as `williams1959` (see Equation 12 above). The `hendrickson1970` formula can be found in Hendrickson et al. [13, p 193], May and Hittner [22, p 259] [23, p 480], and Hittner et al. [11, p 152]:

$$t = \frac{(r_{jk} - r_{jh})\sqrt{(n-3)(1+r_{kh})}}{\sqrt{2|R| + \frac{(r_{jk}-r_{jh})^2(1-r_{kh})^3}{4(n-1)}}}, \quad (18)$$

with $df = n - 3$. A slightly changed version of this formula was provided by Dunn and Clark [25, p 905, formula 1.2], but seems to be erroneous, due to an error in the denominator.

steiger1980: Steiger's [8] modification of Dunn and Clark's [24] z using average correlations

This test was proposed by Steiger [8] and is a modification of Dunn and Clark's [24] z . Instead of r_{jk} and r_{jh} , the mean of the two is used. The test statistic z is defined as

$$z = \frac{(Z_{jk} - Z_{jh})\sqrt{n-3}}{\sqrt{2-2c}} \quad (19)$$

[8, p 247, formula 14], where

$$\bar{r} = \frac{r_{jk} + r_{jh}}{2} \quad (20)$$

[8, p 247] and

$$c = \frac{r_{kh}(1-2\bar{r}^2) - \frac{1}{2}\bar{r}^2(1-2\bar{r}^2 - r_{kh}^2)}{(1-\bar{r}^2)^2} \quad (21)$$

[8, p 247, formula 10; in the original article, there are brackets missing around the divisor].

meng1992: Meng, Rosenthal, and Rubin's [26] z

This test is based on the test statistic z ,

$$z = (Z_{jk} - Z_{jh})\sqrt{\frac{n-3}{2(1-r_{kh})h}} \quad (22)$$

[26, p 173, formula 1], where

$$h = \frac{1 - \bar{r}^2}{1 - r^2} \quad (23)$$

[26, p 173, formula 2],

$$f = \frac{1 - r_{kh}}{2(1 - r^2)} \quad (24)$$

(f must be ≤ 1) [26, p 173, formula 3], and

$$\bar{r}^2 = \frac{r_{jk}^2 + r_{jh}^2}{2} \quad (25)$$

[26, p 173]. This test also constructs a confidence interval of the difference between the two correlation coefficients r_{jk} and r_{jh} :

$$L, U = Z_{jk} - Z_{jh} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{2(1 - r_{kh})h}{n - 3}} \quad (26)$$

[26, p 173, formula 4]. α denotes the desired alpha level of the confidence interval. If the confidence interval includes zero, the null hypothesis that the two correlations are equal must be retained. If the confidence interval does not include zero, the null hypothesis has to be rejected.

hittner2003: Hittner, May, and Silver's [11] modification of Dunn and Clark's [24] z using a backtransformed average Fisher's [2] Z procedure

The approach to backtransform averaged Fisher's [2] Z s was first proposed by Silver and Dunlap [27] and was applied to the comparison of overlapping correlations by Hittner et al. [11]. The test is based on Steiger's [8] approach. The test statistic z is calculated as

$$z = \frac{(Z_{jk} - Z_{jh})\sqrt{n - 3}}{\sqrt{2 - 2c}} \quad (27)$$

[11, p 153], where

$$c = \frac{r_{kh}(1 - 2\bar{r}_z^2) - \frac{1}{2}\bar{r}_z^2(1 - 2\bar{r}_z^2 - r_{kh}^2)}{(1 - \bar{r}_z^2)^2} \quad (28)$$

[11, p 153],

$$\bar{r}_z = \frac{\exp(2\bar{Z} - 1)}{\exp(2\bar{Z} + 1)} \quad (29)$$

[27, p 146, formula 4], and

$$\bar{Z} = \frac{Z_{jk} + Z_{jh}}{2} \quad (30)$$

[27, p 146].

zou2007: Zou's [6] confidence interval

This test calculates the confidence interval of the difference between the two correlation coefficients r_{jk} and r_{jh} . If the confidence interval includes zero, the null hypothesis that the two correlations are equal must be retained. If zero is outside the confidence interval, the null hypothesis has to be rejected. A lower and upper bound for the interval (L and U , respectively) is given by

$$L = r_{jk} - r_{jh} - \sqrt{(r_{jk} - l_1)^2 + (u_2 - r_{jh})^2 - 2c(r_{jk} - l_1)(u_2 - r_{jh})} \quad (31)$$

and

$$U = r_{jk} - r_{jh} + \sqrt{(u_1 - r_{jk})^2 + (r_{jh} - l_2)^2 - 2c(u_1 - r_{jk})(r_{jh} - l_2)} \quad (32)$$

[6, p 409], where

$$l = \frac{\exp(2l') - 1}{\exp(2l') + 1}, \quad (33)$$

$$u = \frac{\exp(2u') - 1}{\exp(2u') + 1} \quad (34)$$

[6, p 406],

$$c = \frac{(r_{kh} - \frac{1}{2}r_{jk}r_{jh})(1 - r_{jk}^2 - r_{jh}^2 - r_{kh}^2) + r_{kh}^3}{(1 - r_{jk}^2)(1 - r_{jh}^2)} \quad (35)$$

[6, p 409], and

$$l', u' = Z \pm z_{\frac{\alpha}{2}} \sqrt{\frac{1}{n-3}} \quad (36)$$

[6, p 406]. α denotes the desired alpha level of the confidence interval.

Tests for Comparison of Two Nonoverlapping Correlations Based on Dependent Groups

The function `cocor.dep.groups.nonoverlap()` implements tests for the comparison of two nonoverlapping correlations based on dependent groups. In the following, r_{jk} and r_{hm} are the two correlations that are being compared; Z_{jk} and Z_{hm} are their Z transformed equivalents. r_{jh} , r_{kh} , r_{jm} , and r_{km} are the related correlations that are also required. n specifies the size of the group the two correlations are based on.

pearson1898: Pearson and Filon's [7] z

This test was proposed by Pearson and Filon [7, p 262, formula xl]. The formula for the test statistic z is computed as

$$z = \frac{\sqrt{n}(r_{jk} - r_{hm})}{\sqrt{(1 - r_{jk}^2)^2 + (1 - r_{hm}^2)^2 - k}} \quad (37)$$

[28, p 179, formula 1], where

$$\begin{aligned} k = & (r_{jh} - r_{jk}r_{kh})(r_{km} - r_{kh}r_{hm}) + (r_{jm} - r_{jh}r_{hm})(r_{kh} - r_{jk}r_{jh}) \\ & + (r_{jh} - r_{jm}r_{hm})(r_{km} - r_{jk}r_{jm}) + (r_{jm} - r_{jk}r_{km})(r_{kh} - r_{km}r_{hm}) \end{aligned} \quad (38)$$

[28, p 179, formula 2]. The two formulae can also be found in Steiger [8, p 245, formula 2 and p. 246, formula 5].

dunn1969: Dunn and Clark's [24] z

The test statistic z of this test is calculated as

$$z = \frac{(Z_{jk} - Z_{hm})\sqrt{n-3}}{\sqrt{2-2c}} \quad (39)$$

[24, p 370, formula 15], where

$$\begin{aligned} c = & \left(\frac{1}{2}r_{jk}r_{hm}(r_{jh}^2 + r_{jm}^2 + r_{kh}^2 + r_{km}^2) + r_{jh}r_{km} + r_{jm}r_{kh} \right. \\ & \left. - (r_{jk}r_{jh}r_{jm} + r_{jk}r_{kh}r_{km} + r_{jh}r_{kh}r_{hm} + r_{jm}r_{km}r_{hm}) \right) \\ & / \left((1 - r_{jk}^2)(1 - r_{hm}^2) \right) \end{aligned} \quad (40)$$

[24, p 368, formula 9].

steiger1980: Steiger's [8] modification of Dunn and Clark's [24] z using average correlations

This test was proposed by Steiger [8] and is a modification of Dunn and Clark's [24] z . Instead of r_{jk} and r_{hm} the mean of the two is being used. The test statistic z is given by

$$z = \frac{(Z_{jk} - Z_{hm})\sqrt{n-3}}{\sqrt{2-2c}} \quad (41)$$

[8, p 247, formula 15], where

$$\bar{r} = \frac{r_{jk} + r_{hm}}{2} \quad (42)$$

[8, p 247] and

$$c = \left(\frac{1}{2}\bar{r}^2(r_{jh}^2 + r_{jm}^2 + r_{kh}^2 + r_{km}^2) + r_{jh}r_{km} + r_{jm}r_{kh} \right. \\ \left. - (\bar{r}r_{jh}r_{jm} + \bar{r}r_{kh}r_{km} + r_{jh}r_{kh}\bar{r} + r_{jm}r_{km}\bar{r}) \right) \\ \left/ (1 - \bar{r}^2)^2 \right. \quad (43)$$

[8, p 247, formula 11; in the original article, there are brackets missing around the divisor].

raghunathan1996: Raghunathan, Rosenthal, and Rubin's [28] modification of Pearson and Filon's [7] z

This test of Raghunathan et al. [28] is based on Pearson and Filon's [7] z . Unlike Pearson and Filon [7], Raghunathan et al. [28] use Z transformed correlation coefficients. The test statistic z is computed as

$$z = \sqrt{\frac{n-3}{2}} \frac{Z_{jk} - Z_{hm}}{\sqrt{1 - \frac{k}{2(1-r_{jk}^2)(1-r_{hm}^2)}}} \quad (44)$$

[28, p 179, formula 3], where

$$k = (r_{jh} - r_{jk}r_{kh})(r_{km} - r_{kh}r_{hm}) + (r_{jm} - r_{jh}r_{hm})(r_{kh} - r_{jk}r_{jh}) \\ + (r_{jh} - r_{jm}r_{hm})(r_{km} - r_{jk}r_{jm}) + (r_{jm} - r_{jk}r_{km})(r_{kh} - r_{km}r_{hm}) \quad (45)$$

[28, p 179, formula 2].

silver2004: Silver, Hittner, and May's [29] modification of Dunn and Clark's [24] z using a backtransformed average Fisher's [2] Z procedure

The approach to backtransform averaged Fisher's [2] Z s was first proposed in Silver and Dunlap [27] and was applied to the comparison of nonoverlapping correlations by Silver et al. [29]. The test is based on Steiger's [8] approach. The formula of the test statistic z is given by

$$z = \frac{(Z_{jk} - Z_{hm})\sqrt{n-3}}{\sqrt{2-2c}} \quad (46)$$

[29, p 55, formula 5], where

$$c = \left(\frac{1}{2}\bar{r}_z^2(r_{jh}^2 + r_{jm}^2 + r_{kh}^2 + r_{km}^2) + r_{jh}r_{km} + r_{jm}r_{kh} - (\bar{r}_z r_{jh}r_{jm} + \bar{r}_z r_{kh}r_{km} + r_{jh}r_{kh}\bar{r}_z + r_{jm}r_{km}\bar{r}_z) \right) / (1 - \bar{r}_z^2)^2 \quad (47)$$

[29, p 56],

$$\bar{r}_z = \frac{\exp(2\bar{Z} - 1)}{\exp(2\bar{Z} + 1)} \quad (48)$$

[27, p 146, formula 4], and

$$\bar{Z} = \frac{Z_{jk} + Z_{hm}}{2} \quad (49)$$

[29, p 55].

zou2007: Zou's [6] confidence interval

This test calculates the confidence interval of the difference between the two correlations r_{jk} and r_{hm} . If the confidence interval includes zero, the null hypothesis that the two correlations are equal must be retained. If the confidence interval does not include zero, the null hypothesis has to be rejected. A lower and upper bound for the interval (L and U , respectively) is given by

$$L = r_{jk} - r_{hm} - \sqrt{(r_{jk} - l_1)^2 + (u_2 - r_{hm})^2 - 2c(r_{jk} - l_1)(u_2 - r_{hm})} \quad (50)$$

and

$$U = r_{jk} - r_{hm} + \sqrt{(u_1 - r_{jk})^2 + (r_{hm} - l_2)^2 - 2c(u_1 - r_{jk})(r_{hm} - l_2)} \quad (51)$$

[6, pp 409–410], where

$$l = \frac{\exp(2l') - 1}{\exp(2l') + 1}, \quad (52)$$

$$u = \frac{\exp(2u') - 1}{\exp(2u') + 1} \quad (53)$$

[6, p 406],

$$c = \left(\frac{1}{2} r_{jk} r_{hm} (r_{jh}^2 + r_{jm}^2 + r_{kh}^2 + r_{km}^2) + r_{jh} r_{km} + r_{jm} r_{kh} \right. \\ \left. - (r_{jk} r_{jh} r_{jm} + r_{jk} r_{kh} r_{km} + r_{jh} r_{kh} r_{hm} + r_{jm} r_{km} r_{hm}) \right) \\ \left/ \left((1 - r_{jk}^2)(1 - r_{hm}^2) \right) \right. \quad (54)$$

[6, p 409], and

$$l', u' = Z \pm z_{\frac{\alpha}{2}} \sqrt{\frac{1}{n-3}} \quad (55)$$

[6, p 406]. α denotes the desired alpha level of the confidence interval.

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